Truthmakers and Information States

Inclusion, Containment, Duality

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ILLC and Philosophy, University of Amsterdam

Prague, November 18, 2025

Workshop on Truthmakers, Possibilities, and Information States

Plan for the talk

I'll discuss a cluster of observations on points of contact between truthmaker and information semantics. These fall under three connected themes:

- Information states (à la BSML) and Containment.
- Truthmakers and Inclusion.
- Translations.

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In BSML, like in inquisitive semantics, formulas are evaluated at sets of valuations ('teams') $t \subseteq \{v \mid v : \mathbf{At} \to \{0, 1\}\}$, $\quad t \vDash \varphi$.

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Observation 2: Telltale of containment logics

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1. Points of contact between BSML and truthmaker semantics.
2. BSML-style information semantics for containment logics.

Semantics for containment logics.

Containment and relevance

Containment logics obey the the proscriptive principle:

$$\varphi \vdash \psi \qquad \text{implies} \qquad \mathbf{At}(\varphi) \supseteq \mathbf{At}(\psi).$$

Strong form of variable sharing:

$$\varphi \vdash \psi \qquad \text{implies} \qquad \mathbf{At}(\varphi) \cap \mathbf{At}(\psi) \neq \varnothing.$$

Signature invalidities:

1. $p \wedge \neg p \nvdash q$ [like relevant logics]
2. $p \nvdash q \vee \neg q$ [like relevant logics]
3. $p \nvdash p \vee q$ [like BSML]

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One prominent containment logic is Angell's analytic entailment AC.
AC is, as shown by Ferguson (2016) and Fine (2016), the containment fragment of FDE:

$$\varphi \vdash_{AC} \psi \qquad \text{iff} \qquad \varphi \vdash_{FDE} \psi \text{ and } \mathbf{Lit}(\varphi) \supseteq \mathbf{Lit}(\psi).$$

Of interest to us because:

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First goal: BSML-style semantics for AC.

BSML and classicality

Recall the BSML semantics: for $t \in \mathcal{P}(\{v \mid v : \mathbf{At} \to \{0,1\}\})$ we define

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Problem: $p \wedge \neg p \vDash q$.

BSML and classicality

Four-valued BSML semantics: for $t \in \mathcal{P}(\{v \mid v : \mathbf{At} \to \mathcal{P}(\{0,1\})\})$ we define

$t \vDash p$	iff	$\forall v \in t, v(p) \ni 1$
$t \mathrel{=\!\!\mid} p$	iff	$\forall v \in t, v(p) \ni 0$
$t \vDash \neg\varphi$	iff	$t \mathrel{=\!\!\mid} \varphi$
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Problem solved: $p \wedge \neg p \nvDash q$. ✓

BSML-style semantics for AC

FDE semantics: Given $\mathcal{P}(X)$, $V^+, V^- : \mathbf{At} \to \mathcal{P}\mathcal{P}(X)$ s.t.

- $V^+(p)$ is a non-empty ideal;
- $V^-(p)$ is a non-empty ideal,

we define for $t \in \mathcal{P}(X)$

$t \vDash p$	iff	$t \in V^+(p)$
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Theorem (FDE completeness)

$\varphi \vDash \psi$ if and only if $\varphi \vdash_{FDE} \psi$.

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AC semantics: Given $\mathcal{P}(X)$, $V^+, V^- : \mathbf{At} \to \mathcal{PP}(X)$ s.t.

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BSML-style semantics for AC

Four-val. BSML* semantics: Given $\mathcal{P}(X)$, $V^+, V^- : \mathbf{At} \to \mathcal{PP}(X)$ s.t.

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Theorem (Four-val. BSML* completeness)

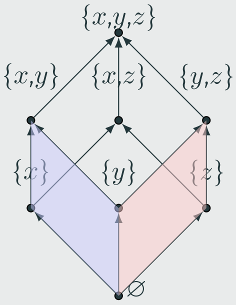
$\varphi \vDash \psi$ *if and only if* $\varphi \vDash_{BSML^\star} \psi$.

FDE, AC, and BSML*

FDE

Always: $[\![p]\!] = \mathcal{I} \ni \varnothing$.

Example:



$$[\![p]\!] = blue;$$

$$[\![\neg p]\!] = red.$$

AC

Possibly: $[\![p]\!] = \mathcal{I}$.

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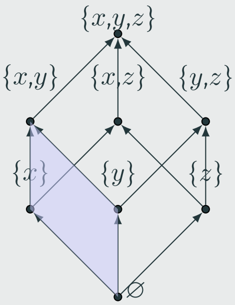

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BSML*

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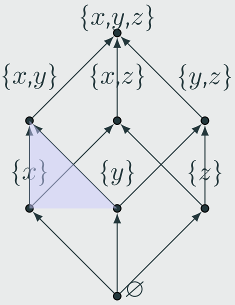

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$$\varphi \vdash_{FDE} \psi \text{ and } \mathbf{At}(\varphi) \supseteq \mathbf{At}(\psi)?$$ [1]

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Proof. As before (note available).

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Follow-ups:

- What other containment logics arise by varying the frames (lattices, semilattices, distributive semilattices, etc.) or valuations?

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Question: As AC is characterized by

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can we also give semantics for the logic characterized by

$$\varphi \vdash_{FDE} \psi \text{ and } \mathbf{At}(\varphi) \supseteq \mathbf{At}(\psi)?$$[1]

Theorem

Require $V^+(p) \neq \varnothing \Leftrightarrow V^-(p) \neq \varnothing$. Then

$$\varphi \vDash \psi \quad \text{iff} \quad \varphi \vdash_{FDE} \psi \text{ and } \mathbf{At}(\varphi) \supseteq \mathbf{At}(\psi).$$

Proof. As before (note available). [Proofs work for distr. lattices too.]

Follow-ups:

- What other containment logics arise by varying the frames (lattices, semilattices, distributive semilattices, etc.) or valuations?
- For instance, can we obtain a complete semantics for Correia's (2016) logic of factual equivalence?

[1] Daniels (1990); Priest (2010).

Recall

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Inferential patterns:

$$p \not\models p \vee q$$

$$p \wedge q \models p$$

Observation 1: Mirror image of truthmaker entailment

Observation 2: Telltale of containment logics

And recall the two guiding themes:

1. Points of contact between BSML and truthmaker semantics.
2. BSML-style semantics for containment logics. ✓

Truthmakers and Inclusion.

Replete truthmaker entailment

Write $\varphi \Vdash \psi$ for replete truthmaker preservation.

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Theorem[2]

Replete truthmaker entailment is the inclusion fragment of FDE; i.e.,

$$\varphi \Vdash \psi \qquad \text{iff} \qquad \varphi \vdash_{FDE} \psi \text{ and } \mathbf{Lit}(\varphi) \subseteq \mathbf{Lit}(\psi).$$

[2] I imagine this is known, but I haven't found it stated.

A sample of corollaries

Corollary

$$\varphi \vdash_{FDE} \psi \text{ and } \mathbf{Lit}(\varphi) = \mathbf{Lit}(\psi) \quad \text{iff} \quad \varphi \vDash \psi \text{ and } \neg\psi \vDash \neg\varphi$$
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Before we proceed, two further remarks on truthmakers and inclusion.

Maxim: *Exactify!*

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Remark 1: On what it means for a semantics to be *exact*.

When is a semantics *exact*?

- Say that $\vDash$ satisfies the inclusion principle if

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- Caveat 1: $\varphi \wedge (\varphi \to \psi) \Vdash \psi$ only when $\mathbf{At}(\varphi) \subseteq \mathbf{At}(\psi)$?
- Caveat 2: How about explosion and its dual? Perhaps inclusion *modulo* explosion and its dual?[3]

[3] The signature invalidities of 'inclusion logics' include explosion and its dual, but maybe exactness should only generalize the invalidity of simplification (think counterfactuals, modalities, etc.).

Remark 2: On replete entailment and wholly relevance.

A-B Analysis: Replete Entailment and Wholly Relevance

Recall Anderson and Belnap's (1962) tautological entailments:

1) For A_i a conjunction of literals, and B_j a disjunction of literals, let

$$A_i \vdash_T B_j \qquad :\text{iff} \qquad \mathbf{Lit}(A_i) \cap \mathbf{Lit}(B_j) \neq \varnothing.$$

2) Lift it as follows:

$$\bigvee A_i \vdash_T \bigwedge B_j \qquad :\text{iff} \qquad \forall i, j : A_i \vdash_T B_j.$$

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1. Like replete entailment, can other truthmaker entailments be given a double-barreled analysis?
2. For instance, can (non-)inclusive entailment be captured by stronger inclusion principles?
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Translations.

Source logic: BSML with NE and ⧫

Fix a non-empty finite set of propositional variables $\mathbf{At}$, and define:

$$\varphi ::= \bot \mid \mathrm{NE} \mid p \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \blacklozenge\varphi.$$

Definition

For $t \subseteq \{v \mid v : \mathbf{At} \to \{0,1\}\}$, we have

$t \models \mathrm{NE}$	iff	$t \neq \varnothing$
$t \mathrel{=\!\!\mid} \mathrm{NE}$	iff	$t = \varnothing$
$t \models \blacklozenge\varphi$	iff	$\exists\, s \subseteq t$ such that $\varnothing \neq s \models \varphi$
$t \mathrel{=\!\!\mid} \blacklozenge\varphi$	iff	$\forall s \subseteq t$: $s \mathrel{=\!\!\mid} \varphi$
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Target logic: modal information logic

Target logic is the modal logic in the language with two modalities,

$$\varphi ::= \bot \mid p \mid \neg\varphi \mid \varphi \vee \varphi \mid \langle \sup \rangle \varphi\varphi \mid \langle \mathsf{s}^* \rangle \varphi,$$

for $p \in \mathbf{At}_{\pm} := \{p_+, p_- \mid p \in \mathbf{At}\}$, interpreted over distributive semilattices $(S, \curlyvee)$, where

$$\begin{aligned} s \Vdash \langle \sup \rangle \varphi\psi \quad &\text{iff} \quad \exists\, t, u \text{ s.t. } t \Vdash \varphi,\ u \Vdash \psi,\ \text{and } s = t \curlyvee u. \\ s \Vdash \langle \mathsf{s}^* \rangle \varphi \quad &\text{iff} \quad \exists\, s_1, \ldots, s_n \text{ s.t. each } s_i \Vdash \varphi \text{ and } s = s_1 \curlyvee \cdots \curlyvee s_n. \end{aligned}$$

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Set

$$\Gamma := \{\mathsf{H}(\mathsf{NE}^+ \vee \mathsf{NE}^-), \bigwedge_{p \in \mathbf{At}} \langle \sup \rangle p^+ p^- \},$$

and define $\cdot^+, \cdot^-$ via the double-recursive clauses:

$$\begin{array}{rcl@{\qquad}lcl}
\bot^+ & := & \mathsf{NE}^- & \bot^- & := & \top \\
\mathsf{NE}^+ & := & \bigwedge_{p \in \mathbf{At}} \neg(p^+ \wedge p^-) & \mathsf{NE}^- & := & \bigwedge_{p \in \mathbf{At}} (p^+ \wedge p^-) \\
p^+ & := & \mathsf{H}\langle \mathsf{s}^* \rangle p_+ & p^- & := & \mathsf{H}\langle \mathsf{s}^* \rangle p_- \\
(\neg\varphi)^+ & := & \varphi^- & (\neg\varphi)^- & := & \varphi^+ \\
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Theorem

$$\varphi \vDash \psi \qquad \text{iff} \qquad \Gamma, \varphi^+ \Vdash \psi^+.$$

Translating BSML

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BSML translation contra truthmaker translation

Translation clauses for BSML:

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(p)^+ &= \mathsf{H}\langle \mathsf{s}^* \rangle p_+ & \quad (p)^- &= \mathsf{H}\langle \mathsf{s}^* \rangle p_- \\
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Translation clauses for truthmaker semantics:[5]

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Translating inquisitive logic

For the case of inquisitive logic, translate $\mathbin{\rotatebox[origin=c]{-90}{$\geqslant$}}$, $\to$ as follows:

$$(\varphi \vee\!\!\!\vee \psi)^+ := \varphi^+ \vee \psi^+$$
$$(\varphi \to \psi)^+ := \mathsf{H}(\varphi^+ \to \psi^+).$$

Theorem (translation of Inq)

$$\varphi \models \psi \qquad \text{iff} \qquad \Gamma, \varphi^+ \Vdash \psi^+.$$

Remark

The translation can be extended to other propositional team logics too, including all fragments of the grammar:

$$\varphi ::= \bot \mid \mathsf{NE} \mid p \mid \neg\alpha \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \blacklozenge\varphi \mid \varphi \vee\!\!\!\vee \varphi \mid \varphi \to \varphi \mid {\sim}\varphi \mid$$
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Thank you!

On connectives:

- *Fact 1:* Team semantics for $\{\neg, \wedge, \vee\}$ gives us classical logic.
- *Fact 2:* In classical logic, $\{\neg, \wedge, \vee\}$ is famously functionally complete: all other connectives are definable by these.
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- *Consequence:* We have a semantic framework for expressions beyond classical assertions, such as questions.

Take-away: *Teams provide for ways to express meanings not readily expressible in single-valuation semantics;* and thus for considering new connectives!

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(Propositional) team logics: propositionhood

- Given any condition-based semantics, we obtain a notion of propositionhood defined as a set of conditions. *Slogan:* Proposition = a set of conditions.
- In team semantics, conditions are teams.
- So, propositions are sets of teams. **Caveat:** The standard terminology is not 'propositions' but 'properties'.

Since our meaning space now has structure (as powersets), we can consider natural restrictions on what a proposition is. Or what different kinds of propostions/meanings there are! For instance, assertions contra questions. (Note the analogy with generalized quantifiers.)

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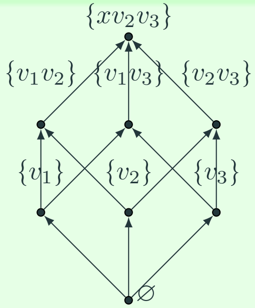

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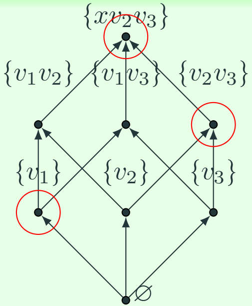

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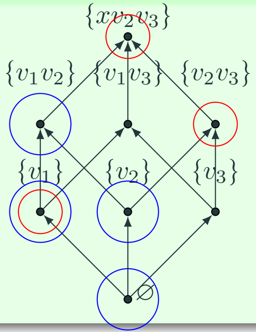

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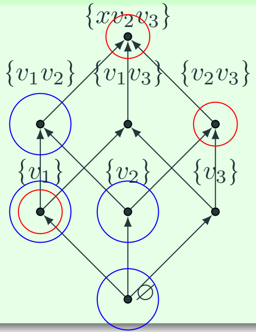

Since our meaning space now has structure (as powersets), we can consider natural restrictions on what a proposition is. Or what different kinds of propostions/meanings there are! For instance, assertions contra questions. (Note the analogy with generalized quantifiers.)

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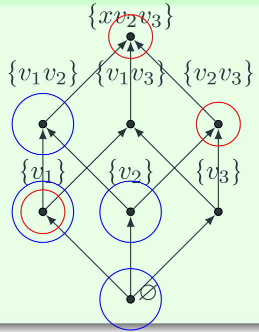

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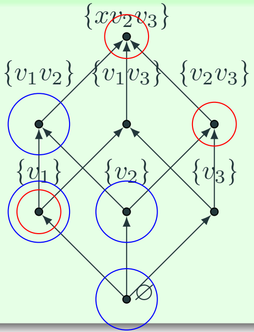

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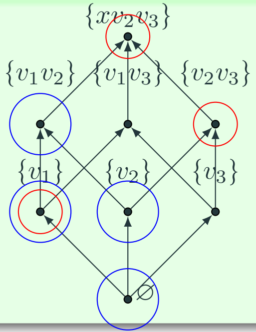

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Notions of propositionhood (closure properties)

Take-away: *Teams provide for ways to express meanings not readily expressible in single-valuation semantics; and thus for considering new notions of propositionhood!*

Definition (some restrictions on propositionhood)

ϕ is *downward closed*:	$[s \models \phi$ and $t \subseteq s] \implies t \models \phi$
ϕ is *union closed*:	$[s \models \phi$ f.a. $s \in S \neq \varnothing] \implies \bigcup S \models \phi$
ϕ has the *empty team property*:	$\varnothing \models \phi$
ϕ is *flat*:	$s \models \phi \iff \{v\} \models \phi$ for all $v \in s$
ϕ is *convex*:	$[s \models \phi, u \models \phi, s \subseteq t \subseteq u] \implies t \models \phi$

Convexity generalizes downward closure:

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Interface of connectives and propositionhood

The choice of connectives and the corresponding notion of propositionhood are closely connected. Here are some examples:

- Classical formulas are flat (so union closed) [i.e., classical assertions are flat]
- Formulas with $\mathbin{\rotatebox[origin=c]{-90}{$\geqslant$}}$ need not be union closed [i.e., questions are not union closed]
- Consider the *epistemic might* operator $\blacklozenge$, defined as

$$s \models \blacklozenge\phi \iff \exists t \subseteq s : t \neq \varnothing \;\&\; t \models \phi.$$

  Formulas with $\blacklozenge$ are not downward closed [i.e., epistemic uncertainty is not persistent]

Theorem (Anttila and SBK (under review))

BSML is expressively complete for convex and union-closed properties.

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